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Modal Analysis of Cantilever Beam using Theoretical and Finite Element Method

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ABSTRACT: In most of the engineering structures the study of vibration plays a significant role. Natural frequency and mode shape are two significant parameters associated with a vibrating body. Natural frequencies and mode shapes of structure can be identified by modal analysis. The modal analysis plays a crucial role in the designing of the mechanical system. In this paper, investigation of vibration characteristics of a cantilever beam is carried out. To determine the natural frequency and mode shape of a single rectangular cantilever beam through finite element analysis software is the objective of this paper. The natural frequencies obtained through finite element analysis software are then compared with the natural frequencies obtained through mathematical calculations of Euler's Bernoulli's beam theory. The study of mild steel single rectangular beam in free fixed condition is carried out. Modes and the natural frequencies applicable to it are computed in finite element analysis software. The cantilever beam is designed and analysed in two FEA software. The two FEA software used for modal analysis are ANSYS & ABAQUS. For mathematical calculations Euler's Bernoulli's beam theory is used. The natural frequencies obtained through ANSYS & ABAQUS are compared with natural frequencies obtained through mathematical equation and are found in good agreement.

KEYWORDS: Modal Analysis, Natural Frequency, Mode Shapes, FEA.

I. INTRODUCTION

Vibration problems are often occurred in mechanical structure. Vibration is a time-dependent movement of objects relative to its equilibrium position. Vibrations can cause serious damages to the mechanical structures so they are undesirable. Every structure has its own natural frequencies. Body will vibrate at its maximum amplitude when the natural frequencies of structure match with external excitation force. This is called resonance. Due to resonance body may get damaged. Resonance can be avoided if natural frequencies of materials are known. Another important term used in the vibration of the structure is the mode shape. A mode shape is the deformation pattern acquired by a mechanical structure at the natural frequency. The shape obtained by structure at resonant frequency is described by mode shape. Modal analysis is a study of the dynamic characteristics of a structure or system under vibration and its response. The modal parameters of the structures can be extracted by modal analysis. The modal parameters are the fundamental elements that describe the response of a structure to free vibrations. The main purpose of modal analysis is to study the dynamic properties of structures like natural frequency, damping and mode shapes. So knowing these modal parameters helps to understand the structure's response to free vibration conditions [1-2]. The objective of this study is to find out the natural frequencies and mode shapes of the rectangular cantilever beam. Modal analysis has done in ANSYS & ABAQUS software to find out natural frequency of the system and mode shapes.

In this work a modal analysis of mild steel single rectangular beam in free fixed condition is carried out. In the present paper, examination and comparison for modal parameters of simple rectangular cantilever beams is carried out. The first technique is by mathematical formulation based on the Euler-Bernoulli beam theory. Next, the FEA software ANSYS & ABAQUS are used to predict the natural frequencies and mode shapes. The natural frequencies obtained by mathematical equation and FEA are compared with each other.

II. LITERATURE SURVEY

Supriya Sahu et al analysed the articulated robot having crack and without having crack for the different modes of frequencies on the finite element method (FEM). The model is designed in CATIA V5 with crack and without crack and a finite element model of the robot is established by utilizing the software of ANSYS Workbench [3]. Kumar et al. performed the free vibration study of heavy vehicle gearbox transmission casing using finite element simulation. The effect of mechanical properties of the four materials on natural frequency and mode shapes of heavy vehicle gearbox transmission casing was examined [4]. Ashwani Kumar et al. have considered the vibration problem of the femur bone to determine the natural frequency and mode shape using FEA method. The FEA based computer is used to identify the fracture location of the bone using free-free and fixed-fixed boundary condition [5]. The effect of boundary condition on the natural frequencies of simply-supported beams is studied by Radice J.J [6]. Migbar Assefa Zeleke analyzed machine tool columns for bending, torsional and modal analysis. Also carried out the study of effect of the orientation and aspect ratio of apertures on the static and dynamic rigidity of the machine tool structure [7]. Ramu I et al. propose a finite element approach to compute the natural frequencies and mode shapes of the FGM plate with different boundary conditions and with different power law index values of rectangular FGM plates based on the Kirchhoff plate theory [8]. Seon M. Han et al analysed four models viz. Euler Bernoulli, Rayleigh, shear and Timoshenko for the transversely vibrating uniform beam. The frequency equations for four sets of end conditions: free-free, clamped-clamped, hinged-hinged and clamped-free are obtained. Hamilton's variational principle is used to obtain the equation of motion for each model, and the expressions for boundary condition [9].

It is important to prevent problems occurred in mechanical structure due to vibrations because it can cause structural fatigue and damage. Resonance is a key aspect in dynamic analysis, which may lead to catastrophic failure. The modal parameters are required to avoid occurrence of resonance in structures due to external excitations. Modal analysis is a very helpful contribution in control of many difficulties encountered in practice due to vibration. Modal analysis received wide acceptance in various fields of science, engineering and technology, it has practical applications like investigations related to aeronautical engineering, automobile engineering and mechanical engineering. In the present scenario a cantilever beam with rectangular cross section is considered. The present investigation reports the dynamic characteristics of common structural material i. e. mild steel.

In this paper modal characteristics and harmonic response of cantilever beam are analysed using ANSYS workbench & ABAQUS. The modal analysis is performed to understand the dynamic behaviour of the structure which results in natural frequency and mode shape of the structure. The harmonic analysis is done to find the maximum response of structure at different frequencies by using ANSYS & ABAQUS.

III. METHODOLOGY

A. Material Specifications:- Material selected for modal analysis of rectangular cantilever beam is Mild Steel. The material properties of the mild steel are as shown in Table 1 [10]

Table1. Material properties of Mild Steel

Young's modulus of Mild Steel	$E = 210 \times 10^9 \text{ (N/mm}^2\text{)}$
Poisson's ratio of Mild Steel	$\nu = 0.24$
Density of Mild Steel	$\rho = 7870 \text{ (Kg/m}^3\text{)}$

Dimensions of the beam are taken as shown in Table 2:-

Table2. Dimensions of cantilever beam

Length of beam	$L = 288 \text{ mm}$
Width of beam	$b = 29.07 \text{ mm}$
Thickness of beam	$h = 2.86 \text{ mm}$

B. Euler–Bernoulli beam equation

Euler–Bernoulli beam theory is used to get natural frequency of rectangular cross sectioned cantilever beam subjected to vibration. The Euler-Bernoulli beam theory is the simple but very useful classical beam theory hence most commonly used. It provides realistic approximations for many engineering problems for single plane bending in the x-y plane.

According to Euler-Bernoulli beam theory, the static equilibrium equations are applied to sum the forces and moments that act in the beam to obtain the equation of a clamped-free uniform beam.

The partial differential equation of motion for free bending vibration of a Euler-Bernoulli beam of a constant rectangular cross-section is given by

$$EI \frac{d^4 y(x, t)}{dx^4} + \rho A \frac{d^2 y(x, t)}{dt^2} = 0 \quad (1)$$

where A is the cross-sectional area, I is the moment of inertia of the cross-sectional area, E is the modulus of elasticity, ρ is the density and $y(x, t)$ is the deflection at the axial location x and at the time t. Since the equation of the motion involves a second order derivative with respect to time and a fourth order derivative with respect to position, two initial equations and four boundary conditions are required to find a unique solution for $y(x, t)$. Equation of motion can be solved by using the methods of separation of variables and eigen functions expansion which gives normal mode solution to the above equation [11].

The deflection $y(x, t)$ are separated into two functions as follows:

$$y(x, t) = X(x) T(t) \quad (2)$$

where $y(x, t)$ is known as the normal mode or characteristic function of the beam.

$$\frac{EI}{\rho A} \frac{d^4 X}{dx^4} T(t) - X(x) \frac{d^2 T}{dt^2} = 0 \quad (3)$$

$$c^2 \frac{d^4 X}{dx^4} + \frac{d^2 T}{dt^2} = 0 \quad (4)$$

$$\text{Where } c = \sqrt{\frac{EI}{\rho A}} \quad (5)$$

$$\beta^4 = \frac{\omega^2}{c^2} = \frac{\rho A \omega^2}{EI} \quad (6)$$

where β is the dimensional natural frequency; ω is the natural angular frequency. Natural frequency f_n of n^{th} mode is given as,

$$f_n = \frac{k_n}{2\pi} \sqrt{\frac{EIg}{wl^4}} \quad (7)$$

K_n is a constant depending on the mode number and end conditions of the beam i. e. on the constraints on the body and w is the weight of the beam per unit length.

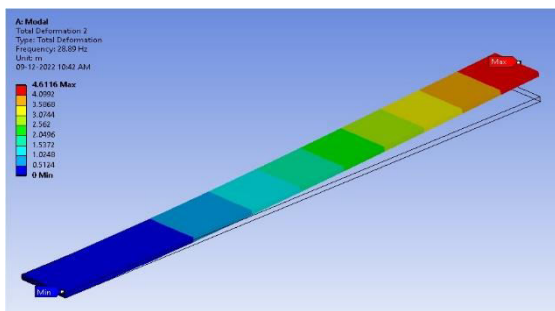
Natural frequencies obtained using equation (7) are compared with the natural frequencies obtained by FEA software as shown in Table 3.

C. Finite Element Method

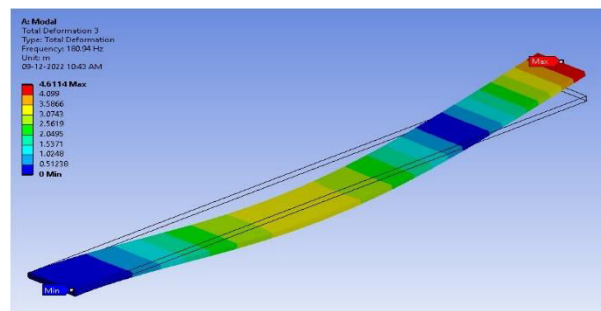
Finite element techniques of analysis and simulation of mechanical systems is used to analyse the static and dynamic behaviour of the structural elements directly on the computer. The finite-element method is a powerful method of analysis that is used to provide approximations to solutions of static and dynamic problems for continuous systems.

FEM divides a large problem into smaller and simpler parts, called finite elements. Using simple equations these finite elements are modelled and assembled into a larger system of equations that will simulate the entire problem. Efficient, general-purpose computer codes exist with suitable matrix assembler and equation solvers to calculate approximate solution for a specific problem.

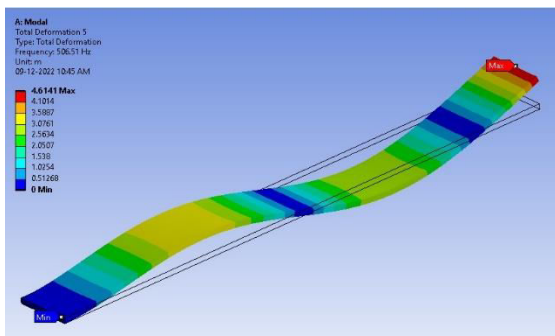
Modal analysis by using ANSYS. ANSYS workbench is used to find out natural frequencies and mode shape of cantilever beam in free fixed condition. Model of single rectangular cantilever beam is constructed in ANSYS and then computational modal analysis is performed to generate natural frequencies and mode shapes. Material properties as mentioned in table 1 are assigned to the model through material library. Boundary condition is applied by providing fixed support at one end of the beam, to calculate natural frequencies and mode shapes. Quadratic meshing type is used in order to split the beam into numerous elements. The number of elements and nodes generated are 4320 and 25037 respectively. Figure 1 shows the different mode shape obtained by modal analysis of a single rectangular cantilever beam in ANSYS.



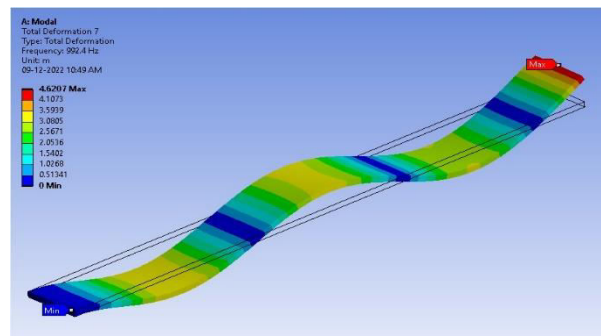
a) First Mode Shape



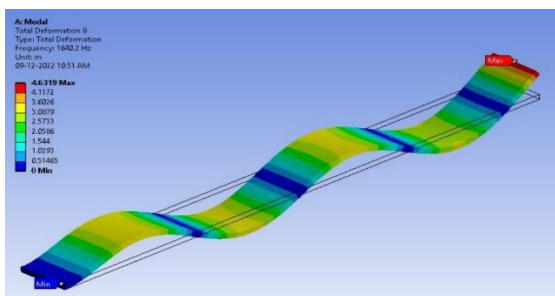
b) Second Mode Shape



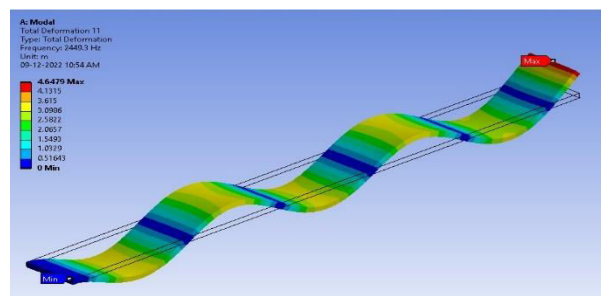
c) Third Mode Shape



d) Fourth Mode Shape



e) Fifth Mode Shape



f) Sixth Mode Shape

Fig. 1. Mode shapes of rectangular cross sectioned cantilever Beam obtained using ANSYS.

Harmonic response Analysis in ANSYS. Harmonic response analysis is used to obtain the frequency response curves of a mechanical system which gives an idea of the peak response of the structure in frequency domain where the excitation is harmonic. The response of the structure is recorded at a given frequency range under a steady state sinusoidal (harmonic) loading. Figure 2 shows directional acceleration of X axis node display of log Y type chart with viewing styles using average spatial resolution. The maximum frequency considered for harmonic analysis in this work is 2505 Hz.

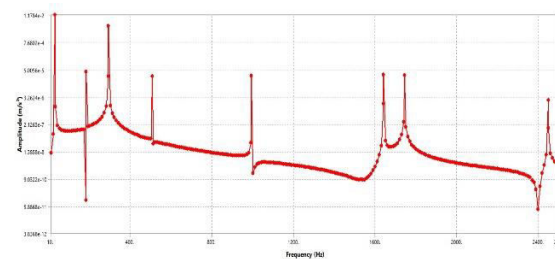
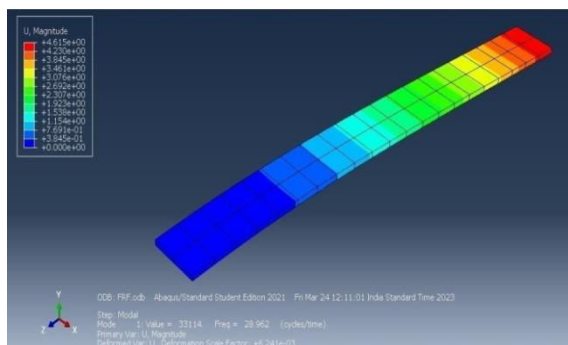
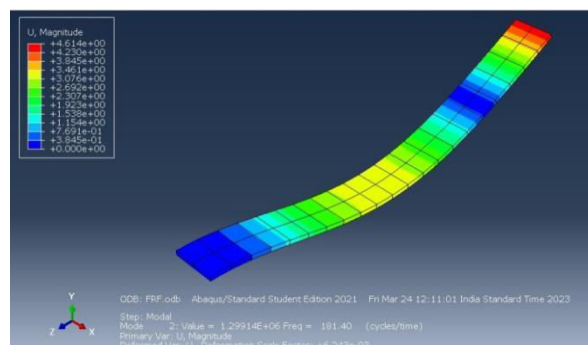


Fig. 2. Directional acceleration of X axis orientation Node display

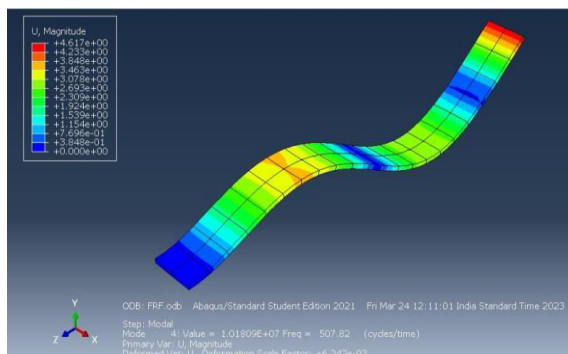
Modal analysis by using ABAQUS. Finite element analysis using ABAQUS has done to obtain the vibration characteristics of the cantilever beam such as natural frequency and mode shapes. Linear perturbation simulation is used to obtain eigen frequencies of the system. Eigen frequencies of the system are obtained in modal analysis step by linear perturbation procedure type using lanczos eigensolver. Encastre constraint on all displacements and rotations is applied on one end of the beam to obtain eigen values and eigen vectors. Standard Quadratic element type is used for meshing. Figure 3 shows the different mode shape obtained by modal analysis of a single rectangular cantilever beam in ABAQUS.



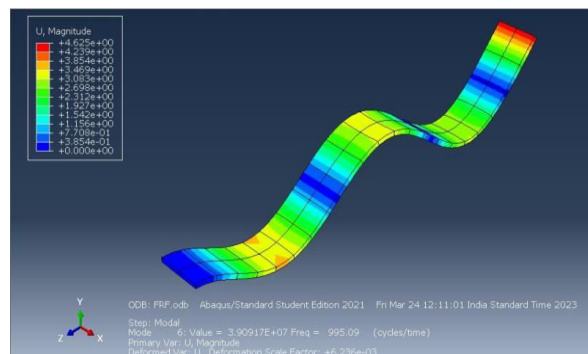
a) First Mode Shape



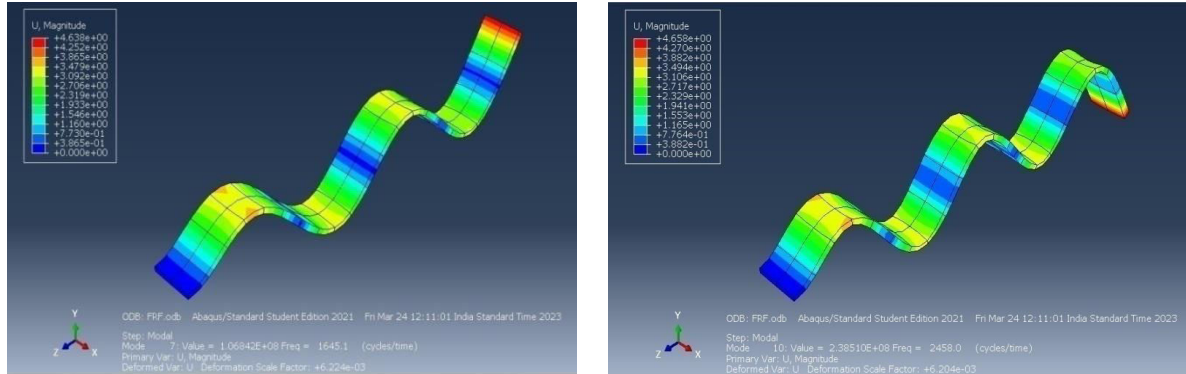
b) Second Mode Shape



c) Third Mode Shape



d) Fourth Mode Shape



e) Fifth Mode Shape

f) Sixth Mode Shape

Fig. 3. Different mode shapes of rectangular cross sectioned cantilever Beam using ABAQUS.

Steady state dynamic analysis. Steady state dynamic, modal procedure type is selected for frequency response function FRF analysis in ABAQUS. Standard quadratic reduced integration element type is assigned to carry out the study of dynamic analysis. The frequency response spectrum obtained as spatial acceleration of X and Y axis is shown in figure 4.

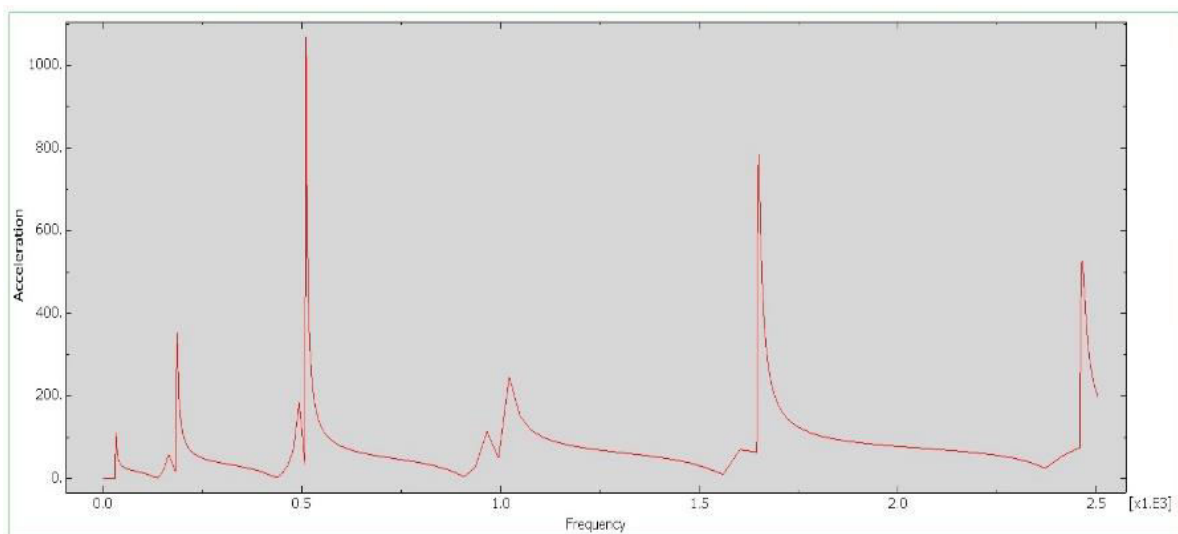


Fig. 4. Spatial acceleration of X axis orientation

IV. RESULTS AND DISCUSSION

In this work, modal analysis of single rectangular cantilever beam has been carried out. Natural frequencies of single rectangular cantilever beam are calculated by equation based on the Euler–Bernoulli beam theory. The theoretical results are compared with the natural frequencies obtained by ANSYS and ABAQUS software. The natural frequencies obtained from beam theory and FEA are compared in table 3. From this analysis it is observed that the natural frequencies obtained are in good agreement with each other.

It can be observed that as the mode order increases, there is greater variation or error between natural frequency obtained by beam theory and natural frequency obtained by software. This may be due to the fact that as frequency is increased, range on which software have to operate rises. As FEA software works on approximate method which in turn increases the chances of error in the measured value over wide operating range.

Table3. Natural frequencies obtained by beam theory and FEA procedure at each mode

Mode	Beam Theory	ANSYS	ABAQUS
1	28.79	28.89	28.96
2	179.97	180.94	181.40
3	504.72	506.51	507.82
4	989.81	992.4	995.09
5	1636.05	1640.2	1645.1
6	2444.30	2449.3	2458.0

The Euler-Bernoulli beam theory gives the bending mode vibration only because it neglects torsion around the longitudinal (x) axis. It can be seen that the natural frequencies obtained by mathematical procedure are in fairly good agreement with the natural frequencies obtained by FEA procedure.

Modal analysis is carried out with the application of the boundary conditions. Mode shapes at different output frequencies for cantilever beam are generated in ANSYS and ABAQUS. The mode shapes generated in ANSYS and ABAQUS for cantilever beam are presented in figure 1 and figure 4 respectively. Figure 1 and figure 4 show the different mode shapes with respect to the corresponding frequency of the beam. Minimum and maximum deflections at various frequencies are observed. The dialog box at the left side contains explicit values of deformations. The lowest value of the deformation was found at the bottom of the beam (Dark blue colour in figure1 and 4) while the maximum (Red colour in figure 1 and figure 4) is at the top of it. From these figures it can be observed that there is significant variation in mode shapes.

V. CONCLUSION

In this paper free vibration analysis of cantilever beam is carried out. Vibration characteristics of a mild steel cantilever beam are investigated. The modal analysis of a cantilever beam with a fixed-free boundary condition is performed. The natural frequencies are obtained using equation of motion and the boundary conditions. Mode shapes of structure at various natural frequencies are determined using finite element analysis technique. An attempt is made to understand the behaviour of the cantilever beam which is subjected to free vibration. The results obtained from Finite element modelling and Euler-Bernoulli equation are found in good agreement. It can be found out that Euler-Bernoulli equation is valid for long and slender beams where we neglect shear deformation effects and torsion.

FEA is a suitable tool for predetermining natural frequencies for design of structure using active vibration control method which helps to modify design or apply some damping material to avoid resonance.

The creation and implementation of an FEA based model can accurately simulate the behaviour of a typical cantilever beam, as long as the necessary requirements (accurate model designs, material properties and boundary conditions) are provided. Boundary condition of the beam plays a very significant role in the vibration response of the beam.

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